Hong Kong Mathematics Olympiad (1997 – 98)

Final Event 1 (Individual)

香港数学竞赛 (1997 – 98)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 a 是 $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$ 的绝对最大值，求 a 的数值。

$a =$

If a is the absolute maximum value of $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$, find the value of a.

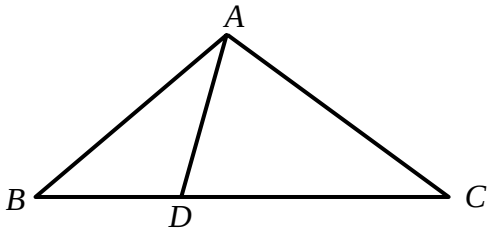
(ii) 若 $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$，求 b 的数值。

$b =$

If $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$, find the value of b.

(iii) 在图中，$BD = b$ cm，$DC = c$ cm，且 ΔABD 的面积 $= \frac{1}{3} \times \Delta ABC$ 的面积，求 c 的数值。

In the figure, $BD = b$ cm, $DC = c$ cm, and the area of $\Delta ABD = \frac{1}{3} \times$ the area of ΔABC, find the value of c.



$c =$

(iv) 设 d 为 $500 + c$ 的正因子的数目，求 d 的数值。

$d =$

Suppose d is the number of positive factors of $500 + c$, find the value of d.

Hong Kong Mathematics Olympiad (1997 – 98)
Final Event 2 (Individual)
香港数学竞赛 (1997 – 98)
决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 $A(1, 3)$、$B(5, 8)$ 及 $C(29, a)$ 共线，求 a 的数值。

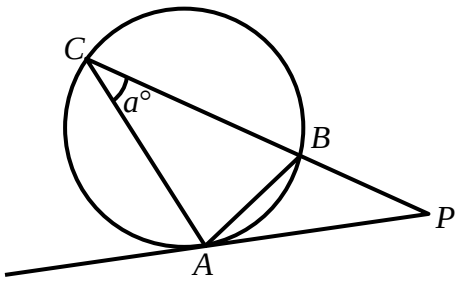
$a =$

If $A(1, 3)$, $B(5, 8)$ and $C(29, a)$ are collinear, find the value of a.

(ii) 在图中，PA 切圆 ABC 于 A。PBC 为一直线、$AB = BP$、$\angle ACB = a°$。若 $\angle ABP = b°$，求 b 的数值。

$b =$

In the figure, PA is a tangent to the circle ABC. PBC is a straight line, $AB = BP$ and $\angle ACB = a°$. If $\angle ABP = b°$, find the value of b.



(iii) 若 c 为二次函数 $y = x^2 + 4x + b$ 的最小值，求 c 的数值。

$c =$

If c is the minimum value of the quadratic function $y = x^2 + 4x + b$, find the value of c.

(iv) 若 $d = 1 - 2 + 3 - 4 + \cdots - c$，求 d 的数值。

$d =$

If $d = 1 - 2 + 3 - 4 + \cdots - c$, find the value of d.

Hong Kong Mathematics Olympiad (1997 – 98)
Final Event 3 (Individual)
香港数学竞赛 (1997 – 98)
决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 $\{p, q\} = q \times a + p$ 且 $\{2, 5\} = 52$，求 a 的数值。

$a =$

If $\{p, q\} = q \times a + p$ and $\{2, 5\} = 52$, find the value of a.

(ii) 若数列 a、$\frac{37}{2}$、b 为一等差数列，求 b 的数值。

$b =$

If a, $\frac{37}{2}$, b is an arithmetic progression, find the value of b.

(iii) 若 $b^2 - c^2 = 200$，求 c 的数值。

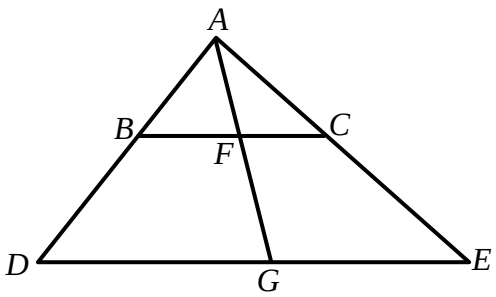
$c =$

If $b^2 - c^2 = 200$, find the value of c.

(iv) 在图中，已知 $BC // DE$、$BC : DE = 10 : c$ 及 $AF : FG = 20 : d$，求 d 的数值。

$d =$

Given that in the figure, $BC // DE$, $BC : DE = 10 : c$ and $AF : FG = 20 : d$, find the value of d.



Hong Kong Mathematics Olympiad (1997 – 98)
Final Event 4 (Individual)
香港数学竞赛 (1997 – 98)
决赛项目 4 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 已知 $\frac{10x-3y}{x+2y}=2$ 且 $p=\frac{y+x}{y-x}$，求 p 的数值。

$p=$

Given that $\frac{10x-3y}{x+2y}=2$ and $p=\frac{y+x}{y-x}$, find the value of p.

(ii) 已知 $a\neq b$ 且 $ax=bx$。若 $p+q=19(a-b)^x$，求 q 的数值。

$q=$

Given that $a\neq b$ and $ax=bx$. If $p+q=19(a-b)^x$, find the value of q.

(iii) 已知 q 个连续数之和为 222，其中最大的是 r，求 r 的数值。

$r=$

Given that the sum of q consecutive numbers is 222, and the largest of these consecutive numbers is r, find the value of r.

(iv) 若 $\tan^2(r+s)^\circ=3$ 且 $0\leq r+s\leq 90$，求 s 的数值。

$s=$

If $\tan^2(r+s)^\circ=3$ and $0\leq r+s\leq 90$, find the value of s.

Hong Kong Mathematics Olympiad (1997 – 98)
Final Event 5 (Individual)
香港数学竞赛 (1997 – 98)
决赛项目 5 (个人)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若方程 $5x^2+ax-2=0$ 的根的和为它的根的积的两倍，求 a 的数值。

$a=$

If the sum of the roots of the equation $5x^2+ax-2=0$ is twice the product of its roots, find the value of a.

(ii) 已知 $y=ax^2-bx-13$ 穿过 $(3, 8)$，求 b 的数值。

$b=$

Given that $y=ax^2-bx-13$ passes through $(3, 8)$, find the value of b.

(iii) 若有 c 种排法把 b 位女孩排成一圆，求 c 的数值。

$c=$

If there are c ways of arranging b girls in a circle, find the value of c.

(iv) 若 $\frac{c}{4}$ 条直线和 3 个圆画于一白纸上，且它们的最多交点数量为 d，求 d 的数值。

$d=$

If $\frac{c}{4}$ straight lines and 3 circles are drawn on a paper, and d is the largest number of points of intersection, find the value of d.